RANDOM WALKS AND TECHNICAL THEORIES: SOME ADDITIONAL EVIDENCE

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I. INTRODUCTION

THE RANDOM WALK and martingale efficient market theories of security price behavior imply that stock market trading rules based solely on the past price series cannot earn profits greater than those generated by a simple buy-and-hold policy[1]. A vast amount of statistical testing of the behavior of security prices indicates very little evidence of any important dependencies in security price changes through time.[2] Technical analysts or chartists, however, have insisted that this evidence does not imply their methods are invalid and have argued that the dependencies upon which their rules are based are much too subtle to be captured by simple statistical tests. In an effort to meet these criticisms Alexander (1961, 1964) and later Fama and Blume (1966) have examined the profitability of various "filter" trading rules based only on the past price series which purportedly capture the essential characteristics of many technical theories. These studies indicate the "filter" rules do not yield profits net of transactions costs which are higher than those earned by a simple buy-and-hold strategy. Similarly, James (1968) and Van Horne and Parker (1967) have found that various trading rules based upon moving averages of past prices do not yield profits greater than those of a buy-and-hold policy.

Robert A. Levy (1967a, b) has reported empirical results of tests of variations of a technical portfolio trading rule variously called the "relative strength" or "portfolio upgrading" rule. The rule is based solely on the past price series of common stocks, and yet his results seem to indicate that some of the variations of the trading rule perform "significantly" better than a simple buy-and-hold strategy. On the basis of this evidence Levy (1967a) concludes that ". . . the theory of random walks has been refuted." In an invited comment Jensen (1967) pointed out that Levy's results do not support a conclusion as strong as this. In that "Comment" it was pointed out that due to several errors the results reported by Levy overstated the excess returns earned by the profitable trading rules over the returns earned by the buy-and-hold comparison. (These arguments will not be repeated here; the interested reader may consult the original articles for the specific criticisms.) Nevertheless, even after correction for these errors Levy's results still indicated some of the trading rules earned substantially more than the buy-and-hold returns, and

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1. Cf. Cootner (1964), Fama (1965), Mandelbrot (1966) and Samuelson (1965).

2. For example, cf. Fama (1965), Roll (1968), and the papers in Cootner (1964).

Jensen (1967) indicated that even these results were inconclusive because of the existence of a subtle form of selection bias.

In his Ph.D. thesis, Levy (1966) reports the results of tests of the profitability of some 68 variations of various trading rules of which very few that were based only on past information yielded returns higher than that given by a buy-and-hold policy.[3] All these rules were tested on the same body of data[4] used in showing the profitability of the additional rules reported by Levy (1967a). Likewise, given enough computer time, we are sure that we can find a mechanical trading rule which "works" on a table of *random numbers*—provided of course that we are allowed to test the rule on the *same* table of numbers which we used to discover the rule. We realize of course that the rule would prove useless on any other table of random numbers, and this is exactly the issue with Levy's results.

As pointed out in the "Comment," the only way to discover whether or not Levy's results are indicative of substantial dependencies in stock prices or are merely the result of this selection bias is to replicate the rules on a different body of data. In a "Reply" Levy (1968) states that additional testing of one of the rules on another body of data[5] yielded returns of 31% per annum. He did not report the buy-and-hold returns for this sample; he did report the returns on the S & P 500-stock index over the same period as slightly less than 10% per annum, and claims the trading rule returns when adjusted[6] to a risk level equal to that of the S & P ". . . would have produced nearly 16% . . .".

The purpose of this paper is to report the results of an extensive set of tests of two of Levy's rules which seemed to earn substantially more than a buy-and-hold policy for his sample of 200 securities in the period 1960-1965.

II. The Trading Rule

The "relative strength" trading rule as defined by Levy is as follows:

Define $\overline{P}_{jt}$ to be the average price of the j'th security over the 27 weeks prior to and including time t. Let $PR_{jt} = P_{jt}/\overline{P}_{jt}$ be the ratio of the price at time t to the 27 week average price at time t. (1) Define a percentage X ($0 < X < 100$) and a "cast out rank" K, and invest an equal dollar amount in the X% of the securities under consideration having the largest ratio PR_{jt} at time t. (2) in weeks $t + \tau$ ($\tau = 1, 2, \ldots$) calculate $PR_{j,t+\tau}$ for all securities, rank them from low to high, and sell all securities currently held with ranks greater than K. (3) Immediately reinvest all proceeds from these sales in the X% of the securities at time $t + \tau$ for which $PR_{j,t+\tau}$ is greatest.

Levy found that the two policies with ($X = 10\%$, $K = 160$) and ($X =$

3. The results for 20 of these rules, none of which show higher returns after transactions costs than the (correct) buy-and-hold returns of 13.4% [cf. Jensen (1967)], are reported in another article by Levy (1967c).

4. Weekly closing prices on 200 securities listed on the New York Stock Exchange in the 5-year period from October, 1960 to October, 1965.

5. The daily closing prices of 625 New York Stock Exchange securities over the period July 1, 1962 to November 25, 1966.

6. No description of his adjustment method was provided.

5%, K = 140) yielded the maximum returns for his sample (20% and 26.1% unadjusted for risk, while the buy-and-hold returns were 13.4%). We have replicated his tests for these two rules for seven non-overlapping 5-year time periods and for 3 to 5 non-overlapping randomly chosen samples of securities within each time period. The results are presented below.

III. The Data

The data for this study were drawn from the University of Chicago Center for Research in Security Prices Monthly Price Relative File.[7] The file contains monthly closing prices, dividends and commission rates on every security on the New York Stock Exchange over the period January, 1926 to March, 1966. In total the file contains data on 1,952 securities and allows one to construct a complete series of (1) dividends and prices adjusted for all capital changes and (2) the actual round lot commission rate on each security for each month.

IV. The Analysis

In order to keep the broad parameters of our replication as close as possible to the original framework used by Levy, we divided the 40-year period covered by our file into the seven non-overlapping time periods (equal in length to Levy's) given in Table 1. (Note that the last period, October 1960-September 1965, is almost identical to Levy's.) After enumerating all securities listed on the N.Y.S.E. at the beginning of *each* of these periods (see Table 1) we randomly ordered them into subsamples of 200 securities each (the same size sample as that used by Levy).

TABLE 1
Sample Intervals and Number of Securities Listed on the N.Y.S.E. at the Beginning of Each Time Period

Time Period*	Number of Securities Listed on N.Y.S.E. at Beginning of Period
(1) Oct. 1930-Sept. 1935	733
(2) Oct. 1935-Sept. 1940	722
(3) Oct. 1940-Sept. 1945	788
(4) Oct. 1945-Sept. 1950	866
(5) Oct. 1950-Sept. 1955	1010
(6) Oct. 1955-Sept. 1960	1044
(7) Oct. 1960-Sept. 1965	1110

* The first 7 months of these periods are used in establishing the initial rankings for the trading rules. Thus the first returns are calculated for May of the following year. All return data are reported for the interval May 1931 through September 1935, etc.

Thus we obtained 29 separate samples of 200 securities each[8] for use in replicating the trading rule—where Levy had one observation on 200 securities we have 29 observations. These 29 independent samples allow us to obtain a very good estimate of the ability of the trading rules to earn profits superior to that of the buy-and-hold policy in any given time period and over

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8. Except for the third time period in which there were only 788 securities listed giving us 4 samples for that time period of 197 securities each.

many different time periods. Note also that we have eliminated one additional source of bias in Levy's procedure by not requiring (as he did) that the securities be listed over the entire 5-year sample period. No investor can possibly accomplish this when actually operating a trading rule since he cannot know ahead of time which firms will stay in business and which will not.

The Trading Profits vs. the B & H Returns.—The average returns earned over all seven time periods for all 29 samples by each of the trading rules and the buy-and-hold (B & H) policy are given in Table 2. The returns on the

TABLE 2
AVERAGE RETURNS AND PERFORMANCE MEASURES OVER ALL PERIODS FOR VARIOUS POLICIES*

Policy	Average Annual Return** Net of Trans. Costs	Average Annual Return** Gross of Trans. Costs	Average Performance Measure $\bar{\delta}$
(1)	(2)	(3)	(4)
Buy-and-Hold***	.107	.111	−.0018
(X = 10%, K = 160)	.107	.125	−.0049
(X = 5%, K = 140)	.093	.124	−.0254

* Calculated over all portfolios in Tables 4 and 5.
** Continuously compounded.
*** Weighted Average. Weights are proportional to number of trading rule portfolios in each period.

B & H policy given in Table 2 are the weighted average returns which would have been earned by investing an equal dollar amount in *every* security listed on the N.Y.S.E. at the beginning of each of the 7 periods under consideration (assuming that all dividends were reinvested in their respective securities when received[9]). The returns net of commissions account for the actual transactions costs involved in the initial purchase and final sale (but ignore the transactions costs on the reinvestment of dividends as do the return calculations on the trading rule portfolios).

We can see from Col. 3 of Table 2 that before allowance for commissions costs the trading rules earned approximately 1.4% more than the B & H policy. However, from Col. 2 of Table 2 we see that after allowance for commissions[10] the trading rules earned returns roughly equivalent to or less than the B & H policy. We shall see below however that the trading rules generate portfolios with greater risk than the B & H policy so that after allowance for the differential risk the rules performed somewhat worse than the B & H

9. If a security was delisted during a particular time period the proceeds were assumed to have been reinvested in the Fisher Investment Performance Index (cf. Fisher [1966]) which was constructed to approximate the returns from a buy-and-hold policy including all securities on the N.Y.S.E.. This procedure is unlikely to cause serious bias and saves a considerable amount of computer time. The weights used in calculating the average B & H returns are proportional to the number of trading rule portfolios in each period. This procedure was followed in order to make the B & H average comparable to the trading rule average in which (due to the differing sample sizes) the time periods receive different weights. The simple averages for each time period are given in Tables 3 and 4.

10. Calculated at the actual round lot rate applying to each security at the time of each trade.

policy. Thus at first glance the results of Levy's trading rule simulation on 200 securities are not substantiated in our replication on 29 independent samples of 200 securities selected over a 35 year time interval.

Fama and Blume (1966) and more recently Smidt (1968) have argued persuasively that these results (the higher returns before allowance for transactions costs and returns comparable or lower than the B & H policy after allowance for transactions costs) are just what we would expect in an efficient market in which traders acting upon information are subject to transactions costs. We can expect outside traders to remove dependencies in security prices only up to the limits imposed by the transactions costs. Any dependencies which are not large enough to yield extraordinary profits after allowance for the costs of acting upon them are thus consistent with the economic meaning of the theory of random walks.

Tables 3 and 4 present the summary statistics of the replication of Levy's trading rules for each time period. Columns 3 and 4 contain the annual returns net and gross of actual transactions costs generated by the trading rule when applied to each sample of 200 securities[11] and for the buy-and-hold comparison. The last line of each panel gives the average values of the trading rule statistics for each sample for the period summarized in the panel.

After transactions costs the (X = 10%, K = 160) trading rule earned more than the B & H policy in only 13 of the 29 cases and the B & H policy showed higher returns in 16 of the 29 cases (see Col. 3 of Table 3). Thus, even ignoring the risk issues, the rule was not able to generate systematically higher returns than the B & H policy. Table 4 shows that the (X = 5%, K = 140) policy performed even less well, yielding a score of 12 to 17 in favor of the B & H policy.

Note also panel 7 of Tables 3 and 4 which gives the results for a time period almost identical to Levy's. The trading rule returns on all 5 portfolios are far smaller than the 20% and 26% respectively he reported. In fact 12.9% is the highest return we obtained in this period and 5 of the 10 rules earned less than the B & H policy. This is additional evidence that Levy's original high returns were spurious and probably attributable to the selection bias discussed earlier.

As before, gross of transactions costs, both trading rules performed much better relative to the B & H policy; with the (X = 10%, K = 160) policy earning higher returns than the B & H policy in 19 of the 29 cases and the (X = 5%, K = 140) policy yielding higher returns in 18 of the 29 cases.

In addition comparison of the mean portfolio return (net of transactions costs) with the B & H return in each subperiod indicates that the B & H returns were higher in 4 out of the 7 periods for the (X = 10%, K = 160) rule and 5 out of the 7 periods for the (X = 5%, K = 140) rule. Gross of transactions costs the B & H policy yielded higher returns in 4 of 7 periods for the (X = 10%, K = 160) policy and 3 of 7 periods for the (X = 5%, K = 140) policy.

11. The data is monthly. Thus the PR_{jt} is defined as the ratio of the price at the end of month t to the average of the closing prices for months t — 6 through month t. The trading rule is then applied at one month intervals for the remainder of the period.

TABLE 3
SUMMARY STATISTICS FOR B & H AND TRADING RULE PORTFOLIOS FOR VARIOUS TIME PERIODS
(Trading Rule is Levy's (X = 10%, K = 160) Policy)

Time Period	Portfolio	Continuously Compounded Annual Rate of Return: Net of Trans. Costs	Continuously Compounded Annual Rate of Return: Gross of Trans. Costs	Std. Dev.*	Beta	Delta
(1)	(2)	(3)	(4)	(5)	(6)	(7)
May 31	B & H	0.047	0.051	0.157	0.942	−0.017
to						
Sep 35	1.	0.088	0.100	0.137	0.774	0.027
[1]	2.	−0.013	0.009	0.112	0.617	−0.066
	3.	−0.032	−0.013	0.151	0.860	−0.093
Portfolio Average		0.014	0.032	0.133	0.750	−0.044
May 36	B & H	−.031	−0.027	0.109	0.929	0.004
to						
Sep 40	1.	−0.081	−0.067	0.095	0.769	−0.057
[2]	2.	−0.048	−0.032	0.106	0.802	−0.020
	3.	−0.103	−0.085	0.104	0.829	−0.078
Portfolio Average		−0.078	−0.062	0.101	0.800	−0.052
May 41	B & H	0.300	0.306	0.058	1.032	−0.043
to						
Sep 45	1.	0.290	0.316	0.059	0.969	−0.032
	2.	0.320	0.347	0.067	1.048	−0.032
[3]	3.	0.237	0.260	0.056	0.881	−0.049
	4.	0.259	0.290	0.071	1.178	−0.116
Portfolio Average		0.277	0.303	0.063	1.019	−0.057
May 46	B & H	0.032	0.036	0.049	0.950	0.012
to						
Sep 50	1.	0.021	0.037	0.055	0.996	−0.000
	2.	0.002	0.019	0.053	0.933	−0.017
[4])	3.	0.031	0.047	0.054	0.983	0.010
	4.	0.006	0.021	0.053	0.952	−0.014
Portfolio Average		0.015	0.031	0.054	0.966	−0.005
May 51	B & H	0.157	0.161	0.031	0.989	−0.004
to						
Sep 55	1.	0.164	0.179	0.039	1.139	−0.016
	2.	0.204	0.219	0.041	1.179	0.013
[5]	3.	0.150	0.170	0.041	1.162	−0.030
	4.	0.162	0.178	0.037	1.026	−0.002
	5.	0.179	0.196	0.033	0.919	0.026
Portfolio Average		0.172	0.188	0.038	1.085	−0.002

* Standard deviation of the monthly returns.

TABLE 3 (Cont'd)

Time Period	Portfolio	Continuously Compounded Annual Rate of Return: Net of Trans. Costs	Continuously Compounded Annual Rate of Return: Gross of Trans. Costs	Std. Dev.	Beta	Delta
(1)	(2)	(3)	(4)	(5)	(6)	(7)
May 56 to	B & H	0.090	0.095	0.033	0.968	0.012
Sep 60	1.	0.272	0.281	0.048	0.829	0.174
	2.	0.125	0.141	0.046	1.067	0.040
[6]	3.	0.110	0.128	0.044	1.122	0.024
	4.	0.201	0.216	0.048	1.096	0.104
	5.	0.083	0.099	0.041	1.076	0.002
Portfolio Average		0.158	0.173	0.045	1.038	0.069
May 61 to	B & H	0.096	0.101	0.039	0.956	0.014
Sep 65	1.	0.129	0.146	0.048	1.044	0.040
	2.	0.087	0.105	0.042	0.922	0.008
[7]	3.	0.101	0.120	0.051	1.161	0.010
	4.	0.063	0.081	0.046	1.032	−0.019
	5.	0.103	0.123	0.044	0.953	0.021
Portfolio Average		0.097	0.115	0.046	1.022	0.012

An Alternative Comparison and a Test of Significance.—Tables 3 and 4 contain the B & H returns calculated for an initial equal dollar investment in *every* security on the exchange at the beginning of each period. We have also calculated the B & H returns which would have been realized on *each sample* of 200 securities. The differences between these B & H returns and the trading rule returns for each sample in each time period are given in Table 5. The results are substantially the same as those reported in Tables 3 and 4 in terms of the number of instances in which the trading rules earned higher returns than the B & H policy (see last two lines of Table 5 for a summary).

The mean difference between the B & H and trading rule returns is given for each policy (both net and gross of transactions costs) in Table 5 along with the standard deviation of the differences. The "t" values given at the bottom of Table 5 (none of which is greater than 1.5) indicate that none of the differences is significantly different from zero. Thus even ignoring the issue of differential risk between the B & H and trading rule policies the trading rules do not earn significantly more than the B & H policy.

V. Risk and the Performance of the Trading Rules

In order to compare the riskiness of the portfolios generated by the trading rules with the risk of the B & H policy we have calculated the standard deviation of the monthly returns (after transactions costs), and these are given in column 5 of Tables 3 and 4. Except for the first two subperiods the standard deviations of the trading rule portfolios are uniformly higher than that for the B & H policy. Thus, for equal expected returns a risk averse

TABLE 4
SUMMARY STATISTICS FOR B & H AND TRADING RULE PORTFOLIOS FOR VARIOUS TIME PERIODS
(Trading Rule is Levy's (X = 5%, K = 160) Policy)

Time Period	Portfolio	Continuously Compounded Annual Rate of Return: Net of Trans. Costs	Continuously Compounded Annual Rate of Return: Gross of Trans. Costs	Std. Dev.	Beta	Delta
(1)	(2)	(3)	(4)	(5)	(6)	(7)
May 31 to	B & H	0.047	0.051	0.157	0.942	−0.017
Sep 35	1.	−0.154	−0.125	0.138	0.728	−0.223
[1]	2.	−0.054	−0.017	0.128	0.672	−0.110
	3.	−0.047	−0.017	0.151	0.822	−01.08
Portfolio Average		−0.085	−0.053	0.139	0.741	−0.147
May 36 to	B & H	−0.031	−0.027	0.109	0.929	0.004
Sep 40	1.	−0.142	−0.121	0.102	0.806	−0.124
[2]	2.	−0.021	0.004	0.141	0.962	0.016
	3.	−0.157	−0.127	0.103	0.761	−0.143
Portfolio Average		−0.107	−0.081	0.116	0.843	−0.083
May 41 to	B & H	0.300	0.306	0.058	1.032	−0.043
Sep 45	1.	0.309	0.352	0.072	1.094	−0.053
	2.	0.326	0.368	0.084	1.160	−0.059
[3]	3.	0.203	0.237	0.066	0.995	−0.110
	4.	0.246	0.292	0.081	1.329	−0.170
Portfolio Average		0.271	0.312	0.076	1.145	−0.098
May 46 to	B & H	0.032	0.036	0.049	0.950	0.012
Sep 50	1.	−0.021	0.005	0.056	1.004	−0.042
	2.	−0.004	0.016	0.056	0.958	−0.024
[4]	3.	0.038	0.060	0.059	1.021	0.017
	4.	−0.003	0.019	0.056	0.965	−0.023
Portfolio Average		0.002	0.025	0.057	0.987	−0.018
May 51 to	B & H	0.157	0.161	0.031	0.989	−0.004
Sep 55	1.	0.155	0.178	0.038	1.074	−0.015
	2.	0.155	0.178	0.042	1.136	−0.023
[5]	3.	0.188	0.213	0.046	1.228	−0.007
	4.	0.132	0.160	0.036	0.949	−0.019
	5.	0.221	0.241	0.039	0.868	0.067
Portfolio Average		0.170	0.194	0.040	1.051	0.001

TABLE 4 (Cont'd)

Time Period	Portfolio	Continuously Compounded Annual Rate of Return: Net of Trans. Costs	Gross of Trans. Costs	Std. Dev.	Beta	Delta
(1)	(2)	(3)	(4)	(5)	(6)	(7)
May 56	B & H	0.090	0.095	0.033	0.968	0.012
to						
Sep 60	1.	0.245	0.258	0.046	0.822	0.152
	2.	0.158	0.181	0.058	1.174	0.064
[6]	3.	0.135	0.159	0.051	1.205	0.043
	4.	0.242	0.263	0.056	1.170	0.135
	5.	0.080	0.106	0.046	1.139	−0.004
Portfolio Average		0.172	0.193	0.052	1.102	0.078
May 61	B & H	0.096	0.101	0.039	0.956	0.014
to						
Sep 65	1.	0.101	0.130	0.053	1.087	0.013
	2.	0.091	0.119	0.047	0.956	0.010
[7]	3.	0.123	0.149	0.060	1.296	0.023
	4.	0.078	0.107	0.053	1.092	−0.009
	5.	0.073	0.104	0.052	1.019	−0.010
Portfolio Average		0.093	0.122	0.053	1.090	0.005

investor choosing among portfolios on the basis of mean and standard deviation would not be indifferent between them. This brings us to a serious issue.

If securities markets are dominated by risk-averse investors and risky assets are priced so as to earn more on average than less risky assets then any portfolio manager or security analyst will be able to earn above average returns if he systematically selects a portfolio with higher than average risk; so too will a mechanical trading rule. Jensen (1967) has pointed out that there is good reason to believe that Levy's trading rules will tend to select such an above average risk portfolio during time periods in which the market is experiencing generally positive returns. Thus it is important in comparing the returns of the trading rule to those of the B & H policy to make explicit allowance for any differential returns due solely to different degrees of risk.

A Portfolio Evaluation Model.—Jensen (1969) has proposed a model for evaluating the performance of portfolios which takes explicit account of the effects of differential riskiness in comparing portfolios. The model is based upon recent mean-variance general equilibrium models of the pricing of capital assets proposed by Sharpe (1964), Lintner (1965), Mossin (1966), and Fama (1968). The measure of performance, δ_j for any portfolio j in any given holding period suggested by Jensen is

$$\delta_j = R_j - [R_F + (R_M - R_F)\beta_j] \quad (1)$$

where

R_j = the rate of return on portfolio j.
R_F = the riskless rate of interest.

R_M = the rate of return on a market portfolio consisting of an investment in each outstanding asset in proportion to its value.

$$\beta_j = \frac{\text{cov}(R_j, R_M)}{\sigma^2(R_M)} = \text{the systematic risk of the j'th portfolio.}$$

We shall not review the details of the derivation of (1) here; the interested reader is referred to Jensen (1969). However, Figure 1 gives a graphical in-

TABLE 5
DIFFERENCES BETWEEN B & H AND TRADING RULE RETURNS.
(B & H RETURNS CALCULATED FOR EACH SUBSAMPLE OF 200 SECURITIES.)

	B & H Returns—Trading Rule Returns			
	[X = 10%, K = 160]		[X = 5%, K = 140]	
Period	Net of Trans. Costs	Gross of Trans. Costs	Net of Trans. Costs	Gross of Trans. Costs
(1)	(2)	(3)	(4)	(5)
	−0.024	−0.032	0.218	0.193
1	0.057	0.040	0.098	0.066
	0.079	0.065	0.094	0.069
	0.035	0.024	0.096	0.078
2	0.033	0.021	0.006	−0.015
	0.074	0.061	0.128	0.103
	0.012	−0.008	−0.007	−0.044
3	−0.013	−0.033	−0.019	−0.054
	0.039	0.021	0.073	0.044
	0.058	0.034	0.071	0.032
	0.012	0.0	0.054	0.032
4	0.030	0.016	0.036	0.019
	0.008	−0.004	0.001	−0.017
	0.020	0.008	0.029	0.010
	−0.016	−0.027	−0.007	−0.026
	−0.032	−0.043	0.017	−0.002
5	0.003	−0.012	−0.035	−0.055
	−0.012	−0.024	0.018	−0.006
	−0.017	−0.029	−0.059	−0.074
	−0.177	−0.181	−0.150	−0.158
	−0.034	−0.045	−0.067	−0.085
6	−0.022	−0.035	−0.047	−0.066
	−0.100	−0.110	−0.141	−0.157
	−0.004	−0.016	−0.001	−0.023
	−0.033	−0.045	−0.005	−0.029
	0.002	−0.011	−0.002	−0.025
7	0.003	−0.011	−0.019	−0.040
	0.035	0.022	0.020	−0.004
	0.005	−0.010	0.035	0.009
Mean Difference = $\bar{d}$	.001	−.013	.015	−.008
Std. Dev. = $\sigma(\tilde{d})$	.050	.048	.075	.072
$t(\bar{d}) = \bar{d}/(\sigma(\tilde{d})/\sqrt{29})$	1.07	−1.46	1.08	−.60
Number (−)	12	18	13	18
Number (+)	17	11	16	11

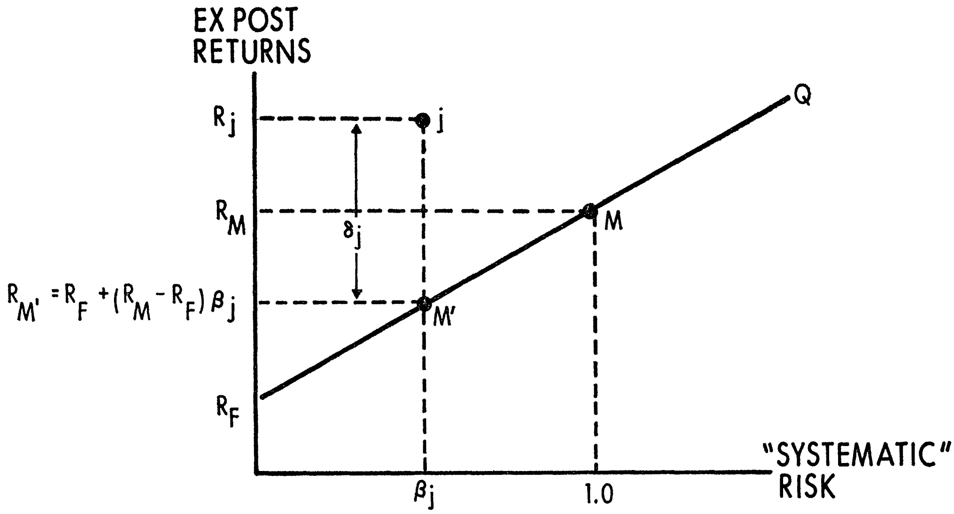

FIGURE 1
The Measure of Performance δ_j, for a Hypothetical Portfolio

terpretation of the measure of performance δ_j. The point M represents the realized returns on the market portfolio and its "systematic" risk (which from the definition of β, can be seen to be unity). The point R_F is the riskless rate and the equation of the line R_FMQ is

$$E(R|R_M, \beta) = R_F + (R_M - R_F)\beta. \quad (2)$$

If the asset pricing model is valid, the line R_FMQ given by eq. (2) gives us the locus of expected returns on any portfolio conditional on the ex post market returns and the systematic risk of the portfolio, β, in the absence of any forecasting ability by the portfolio manager. Thus the line R_FMQ represents the trade off between risk and return which existed in the market over this particular holding period. The point j represents the ex post returns R_j on a hypothetical portfolio j over this holding period, and β_j is its systematic risk. The vertical distance between the risk-return combination of any portfolio j and the line R_FMQ in Figure 1 is the measure of performance of portfolio j.

In the absence of any forecasting ability by the portfolio manager the expected value of δ_j is zero. That is we expect the realized returns of the portfolio to fluctuate randomly about the line R_FMQ through successive holding intervals. If $\delta_j > 0$ systematically, the portfolio has earned returns higher than that implied solely by its level of risk, and therefore the manager can be judged to have superior forecasting ability. If $\delta_j < 0$ systematically, the portfolio has earned returns less than that implied by its level of risk, and if the model is valid this can only be explained by the absence of forecasting ability and the generation of large expenses by the manager (see Jensen [1969, pp. 227f]).

The measure δ_j may also be interpreted in the following manner: Let M′ be a portfolio consisting of a combined investment in the riskless asset and the market portfolio M such that its risk is equal to β_j. Now δ_j may be interpreted as the difference between the return realized on the j'th portfolio and the return $R_{M'}$ which could have been earned on the equivalent risk market portfolio M′. If $\delta_j > 0$, the portfolio j has yielded the investor a return greater than the return on a combined investment in M and F with an identical level of systematic risk.

The measures of systematic risk for each of the portfolios generated by the trading rules and for the B & H policy are given in column 6 of Tables 3 and 4, and the measures of performance δ_j are given in column 7. The market returns and risk free rates used in these estimates are given in Table 6. The

TABLE 6

MARKET AND RISKLESS RETURNS USED IN ESTIMATING THE PERFORMANCE MEASURES δ_j

Period	Market Return*	Riskless Rate**
1) May 1931-Sept. 1935	.064	.0334
2) May 1936-Sept. 1940	−.039	.0108
3) May 1941-Sept. 1945	.296	.0080
4) May 1946-Sept. 1950	.020	.0104
5) May 1951-Sept. 1955	.149	.0206
6) May 1956-Sept. 1960	.075	.0296
7) May 1961-Sept. 1965	.079	.0344

* Continuously compounded returns on Fisher Investment Performance Index (Fisher [1966]), obtained from most recent Monthly Price Relative Tape distributed by Standard Statistics, Inc.

** Continuously compounded yield to maturity (at the beginning of the period) of a government bond maturing at the end of the period estimated from yield curves presented in the U. S. Treasury Bulletin, except for the first two periods. The rate for the first period is the average yield on long-term government bonds at the beginning of the period taken from the *Eighteenth Annual* Report of the Federal Reserve Board—1931 (Washington, D.C., 1932), p. 79. The rate for the second period is the average yield on U.S. Treasury 3-5 year notes taken from the *Twenty-Third Annual Report of the Board of Governors of the Federal Reserve System—1936* (Washington, D.C., 1937), p. 118.

average δ's for the B & H policy and the trading rule portfolios over all periods are given in column 4 of Table 2. The $\bar{\delta}$ for the B & H policy (after transactions costs) over all 7 periods was −.0018; that is the B & H policy earned on average .18% per year (compounded continuously) less than that implied by its level of risk and the asset pricing model.

On the other hand the average δ for the trading rules (net of transactions cost) was −.49% and −2.54% respectively for the ($X = 10\%$, $K = 160$) and ($X = 5\%$, $K = 140$) policies. That is, after explicit adjustment for the systematic riskiness of the two policies, they earned −.49% and −2.54% less than that implied by their level of risk and the asset pricing model. In addition the average δ for the portfolios was greater than the δ for the B & H policy in only 2 periods for both of the trading rules (see Tables 3 and 4). Since the point at issue is whether or not the trading rules perform *significantly better* than the B & H policy the fact that they don't on the average even perform as well means we need not bother with any formal tests of significance.

VI. Summary and Conclusions

Our replication of two of Levy's trading rules on 29 independent samples of 200 securities each over successive 5 year time intervals in the period 1931 to 1965 does not support his results. After allowance for transactions costs the trading rules did not on the average earn significantly more than the B & H policy. Furthermore, since the trading rule portfolios were on the average more risky than the B & H portfolios this simple comparison of returns is biased in favor of the trading rules. After explicit adjustment for the level of risk it was shown that net of transactions costs the two trading rules we tested earned on average —.31% and —2.36% less than an equivalent risk B & H policy. Given these results we conclude that with respect to the performance of Levy's "relative strength" trading rules the behavior of security prices on the N.Y.S.E. is remarkably close to that predicted by the efficient market theories of security price behavior, and Levy's (1967a) conclusion that ". . . the theory of random walks has been refuted," is not substantiated.

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